Quantum Science & Technology
Interdisciplinary Academic Program, IIT Madras

Contents

1 Quantum Science & Technology

1.1 Quantum Computation and Quantum Information

Introduction: Quantum states, density operators, generalized measurements, quantum operations/channels, no-cloning theorem.
Quantum correlations: Bell inequalities and entanglement, Schmidt decomposition, super-dense coding, teleportation, PPT criterion.
Quantum gates and algorithms: Universal set of gates, quantum circuits, Solovay-Kitaev theorem, Deutsch-Josza algorithm, period-finding, factoring, Shor's algorithm, quantum search, Abelian quantum hidden subgroup problem.
Quantum information theory: Shannon entropy, noiseless coding theorem, von Neumann entropy and properties, Schumacher compression, noisy coding theorem.
Quantum cryptography: quantum key distribution, entropic uncertainty relations.
Quantum noise and error-correction: Distance measures, Knill-Laflamme conditions, quantum error-correcting codes, Hamming bound.

1.2 Experimental techniques for Quantum Computation and Metrology

Introduction and basics: Basics of quantum mechanics as needed for the course, basics of common techniques used (e.g.: Magnetic resonance, optics, cryogenics)
Various quantum sensors and qubits technologies/platforms: (emphasis on experimental and device aspects) Superconducting qubits, trapped ions/atoms, photon-based, solid-state spins etc.
Noise, sensitivity and information content: Noise (classical and quantum), imperfections in experiments, SNR, quantum bounds on information etc.

Experimental aspects of sensing and computation protocols: Quantum gates, Ramsey, Rabi and Echo sequences; Dynamic decoupling and multi-pulse protocols; AC signals and quantum lock-in; quantum memory and registers etc.

1.3 Quantum Integer Programming

Offered online by Prof. Sridhar Tayur, CMU, Fall'20
Integer programming (classical methods): Integer Programming basics, Cutting plane theory and relaxations, Introduction to test sets, Gr¨obner basis, Graver basis.
Ising, QUBO: Ising model basics, simulated annealing, Markov-chain Monte Carlo methods, benchmarking classical methods, formulating combinatorial problems as QUBOs
Hardware for solving Ising/QUBO: Graphical Processing Units, Tensor Processing Units, Digital Annealers, Oscillator Based Computing, Coherent Ising Machines,
Quantum methods for solving Ising/QUBO: Adiabatic Quantum Computing and Quantum Annealing, Quantum Approximate Optimization Algorithm
Graver Augmented Multiseed algorithm (GAMA): GAMA, Applications in Portfolio Optimization, Cancer Genomics, Quantum Inspired Quadratic (Semi-)Assignment Problem
Hardware: Compiling, Quantum Annealing, Gate-based Noisy Intermediate Scale Quantum (NISQ) devices

1.4 Quantum Optics

Correlation functions of light waves: Spectral representation of mutual coherence function, calculation of mutual intensity and degree of coherence, propagation of mutual intensity, rigorous theory of partial coherence, coherency matrix of a quasi-monochromatic plane wave, stochastic description of light and higher order coherence effects,
Second quantization: Quantization of the radiation field, quantum mechanical harmonic oscillator, the zero point energy, states of the quantized radiation field, single mode number states and phase states, coherent photon states.
Quantum theory of the laser: photon rate equations, time dependence of photon coherence, laser threshold condition, rate equations for atoms and laser photons, laser photon distribution, fluctuations in laser light and laser phase diffusion.
Statistical optics of photons: Photon coherence properties, photon counting, photon distribution for coherent and chaotic light, quantum mechanical photon counting distribution.
Super radiance: collective cooperative spontaneous radiation, Diecke's theory, Photon echoes, Quantum beats, Quantum chaos and instability hierarchies of laser light, chaos and its routes, Squeezed states of light.

1.5 Quantum Photonics Devices and Technology

Wave Mechanics and Quantum States of Light: Wave Function, Eigen States and Superposition, Concept of Quantum Bit (Qubit), Cavity Quantum Harmonic Oscillator, Photon Statistics, Coherent States and Squeezed Light, Photon Number States
Practical Devices for Quantum Photonic Functions: Generation of Entangled Photon Pairs and Heralded Single Photons, Beam Splitter/Filters, Mach-Zehnder Interferometer, Delay Line and Phase-Shifter, Photon Detection/Counters
Integrated Quantum Photonics: Technology and Platforms, Quantum Random Number Generator (QRNG), On-chip Quantum Key Distribution (QKD), Linear Optical Quantum Computing (LOQC) with Photonic Qubit
Starting Fall 2020, experiments with entangled photons and single photon detectors are credited under **Quantum Electronics & Lasers** *or* **Advanced Photonics Lab.**

1.6 Quantum Mechanics

Angular momentum: Orbital and spin angular momentum, Angular momentum algebra, Eigenstates and eigenvalues of angular momentum, Addition of angular momenta, Clebsch-Gordon coefficients, Irreducible tensor operators and the Wigner-Eckart theorem,
Systems of identical particles: Symmetric and anti-symmetric wavefunctions, Bosons and Fermions, Pauli's exclusion principle, Second quantization, occupation number representation,
Scattering: Non-relativistic scattering theory, Scattering amplitude and cross- section, The integral equation for scattering, Born approximation, Partial wave analysis, The optical theorem,

Relativistic quantum mechanics: The Klein-Gordon equation, the Dirac equation, Dirac matrices, spinors, positive and negative energy solutions, physical interpretation, non-relativistic limit of the Dirac equation.

2 Information Theory and Coding

2.1 Information Theory

Entropy, Relative Entropy, and Mutual Information: Entropy, Joint Entropy and Conditional Entropy, Relative Entropy and Mutual Information, Chain Rules, Data-Processing Inequality, Fano's Inequality.
Typical Sequences and Asymptotic Equipartition Property: Asymptotic Equipartition Property Theorem, Consequences of the AEP: Data Compression, High-Probability Sets and the Typical Set.
Source Coding and Data Compression: Kraft Inequality, Huffman Codes, Optimality of Huffman Codes.
Channel Capacity: Symmetric Channels, Properties of Channel Capacity, Jointly Typical Sequences, Channel Coding Theorem, Fano's Inequality and the Converse to the Coding Theorem.
Differential Entropy and Gaussian Channel: Differential Entropy, AEP for Continuous Random Variables, Properties of Differential Entropy, Relative Entropy, and Mutual Information, Coding Theorem for Gaussian Channels.
Linear Binary Block Codes: Introduction, Generator and Parity-Check Matrices, Repetition and Single-Parity-Check Codes, Binary Hamming Codes, Error Detection with Linear Block Codes, Weight Distribution and Minimum Hamming Distance of a Linear Block Code, Hard-decision and Soft-decision Decoding of Linear Block Codes, Cyclic Codes, Parameters of BCH and RS Codes, Interleaved and Concatenated Codes.

2.2 Stochastic Process:

Fundamental concepts: Random variables Probability distributions, moments and generating functions, central limit theorem, Random events expected number of occurrences, waiting time distributions, Stochastic processes Stationarity, transition probabilities, Markov property, Chapman-Kolmogorov equation.

Discrete Markov processes: The master equation and its solutions, detailed balance, single-step processes- random walks and telegraph process, random walks in the presence of barriers, survival problems and first passage densities,macroscopic equations.
Continuous Markov processes: Kramers-Moyal expansion, Fokker-Planck equation, Langevin equation for Brownian motion, Wiener and Ornstein-Uhlenbeck processes,forward and backward Kolmogorov equations.

2.3 Topics in Random Processes and Concentrations

Review of Basic Probability Theory: Measure Spaces, Sigma-Algebras, Random Variables, Expectation, Convergence Theorems, Conditional Expectation, Filtrations.
Martingales: Definitions, basic properties, sub-martingales, and super-martingales, examples, Doob's decomposition, stopping times, gambling strategy, optional stopping theorem, Martingale convergence theorem, Doob-Kolmogorov's inequality, Random Walks, exchangeability, de Finetti's theorem, Applications to queueing theory, information theory
Large Deviation Theory: Large Deviation for i.i.d. variables, Chernoff Bound, Legendre transform, Cramer's theorem, rate function and its properties, change of measures, Gartner-Ellis theorem, Large Deviation for Markov chains Applications to queueing theory.
Concentration Inequalities: Martingale concentrations (Azuma-Hoeffding, Doob's martingale method, median concentrations), MGF methods, Logarithmic Sobolev inequality, Talagrand's inequality, Hanson-Wright inequality.

2.4 Advanced Statistical Physics:

Collective behavior, from particles to field, phonon-elasticity, Phase transition and critical phenomenon.
Statistics of field: The Landau–Ginzburg Hamiltonian,Saddle point approximation, and mean-field theory,Continuous symmetry breaking and Goldstone modes,Discrete symmetry breaking and domain walls, Scattering and fluctuations, The scaling hypothesis.
Perturbative renormalization group: Expectation values in the Gaussian model, Expectation values in perturbation theory, Diagrammatic representation of perturbation theory (Feynman Diagram), Susceptibility,Perturbative RG (first order, second order), The ϵ expansion, Irrelevance of other interactions.

2.5 Error Control Coding

Mathematical Preliminaries Groups, rings, fields, vector spaces, linear algebra, finite fields: construction, structure of fields, polynomials over finite fields, minimal polynomials, factorization of polynomials
Linear block codes Generator and parity check matrices, dual code, distance of a code. Decoding linear codes: MAP decoder, ML decoder, standard array and syndrome decoding, bounded distance decoder. Bounds on codes: Singleton, Hamming, Plotkin, Gilbert-Varshamov bounds and asymptotic bounds Weight enumerators, MacWilliams relation for binary block codes Code constructions: puncturing, extending, shortening, direct sum, product construction, interleaving, concatenation Performance of block codes
Important algebraic block codes: Cyclic codes, BCH codes, Reed-Solomon codes, Reed-Muller codes and Hamming codes Berlekamp-Massey algorithm for decoding BCH and Reed-Solomon codes
Convolutional codes Various formulations of convolutional codes using shift registers, generator sequences, polynomials, and matrices, recursive and non recursive encoders Code parameters: constraint length, memory, free distance Structural properties of convolutional codes: state diagram, trellis diagram, non-catastrophic encoders, weight enumerators
Decoding convolutional codes: Viterbi and BCJR algorithms, hard decision and soft decision decoding Performance of convolutional codes 5. Capacity achieving codes LDPC codes: Tanner graphs, Low density parity check (LDPC) codes, iterative decoding, bit flipping and sum product algorithms Introduction to turbo codes

3 Photonics

3.1 Optics and Photonics

Fourier Optics: Diffraction integral, Fourier transformation in beam propagation, Fresnel and Fraunhoffer approximations, Fourier filtering, Image processing, principle of phase contrast microscope, holography and principles of recording and reconstruction.
Optics of periodic media: multi-layer dielectric interference coatings and their applications, photonic crystals, Bragg reflectors.
Lasers: optical amplification and lasers, characteristics of laser radiation, spatial and temporal coherence, optics of Gaussian beams.
Fibre and Integrated Optics: Guided modes, attenuation and dispersion in optical fibres, application in sensors and communication, photonic devices based on acousto-optics, electro-optics and magneto-optics,
Introductory treatment of nano-photonics: negative refraction and meta-materials, nonlinear optical processes, slowing of light and other contemporary topics.

3.2 Laser Physics

Characteristics of laser radiation: Coherence and brightness, Einstein's coefficients, rate equations, Gain coefficient, 3, 4- level lasers, Threshold gain, Creation and annihilation operators.
Modes of oscillation: transverse and longitudinal, spectral line shapes and line broadening mechanisms, applications in frequency stabilization.
Gaussian Beams: Stability conditions, Rayleigh Range, confocal parameter.
Typical laser systems: Gas (He-Ne, He-Cd, Argon, CO2, Excimer), Solid (Ruby, Nd:YAG, Ti:Saphire), Fiber lasers, semiconductor, free-electron lasers; Uniqueness and their applications.
General Applications: Laser holography, holographic storage, non-destructive testing, optical disk storage.